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Photometric Observations of ω Centauri: Multi-Wavelength Observations of Evolved Stars

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Abstract. We present multi-wavelength observations of the northern population of ω Cen from the main-sequence turn-off to high on the red giant branch. We show that the best information about the metallicity and age of the stars can be gained from combining vby, B-I and V-I colors (in the absence of spectroscopy). We confirm our results for the main-sequence turn-off stars: there is at least a 3 Gyr age spread, which may be as large as 5-8 Gyr, as suggested by Hughes & Wallerstein (2000) and Hilker & Richtler (2000). We find that B-I colors can be affected by excessive CN-content (which can vary at a given value of [Fe/H]). We use proper motion studies to confirm cluster membership at and above the level of the horizontal branch, and we show that the age spread is maintained amongst stars from the subgiant branch through the red giants. We support previous findings that there is another red giant branch, redder (Pancino et al. 2000), and younger than the main giant branch (Hilker & Richtler 2000) but containing few stars. Even though the spatial distribution of the metal-rich stellar population is different from that of the metal-poor population (which could be achieved by capturing a smaller metal-rich star cluster, as suggested by Pancino et al. 2000), it is likely that ω Cen is the core of a captured dwarf galaxy.

1. Introduction

Why does ω Cen have a spread in metallicity? Can the largest and most massive globular cluster in our galaxy (about $3 \times 10^6 M_\odot$), be a composite object, made up of separate star clusters which merged after they finished forming stars (Norris et al. 1997; Norris et al. 1996; Jurcsik 1998)? One or more mergers of star clusters could give an age spread along with a metallicity spread, but this seems dynamically unlikely (Thurl & Johnston 2000, Thurl 2002). The structure

of ω Cen is also fairly “loose” compared to more centrally condensed clusters like 47 Tuc (Mayor et al. 1997). Bessell & Norris (1976) suggested that the gas cloud which gave birth to ω Cen was chemically inhomogeneous, and Norris et al. (1997) also discussed the possibility the cluster could have formed when cloud fragments with different metallicities mixed. All these scenarios could give rise to the metallicity spread which has been observed by many researchers, but most notably detailed by Smith (1987) and Suntzeff & Kraft (1996).

Currently, the most popular theory for the origin of ω Cen involves the Milky Way capturing and disrupting a dwarf galaxy (possibly a dwarf elliptical like the Sagittarius system), which is dynamically more likely than merging separate clusters (Majewski 1999; Dinescu et al. 1999; Lee et al. 1999). Self-enrichment was considered by Morgan & Lake (1989); their theoretical calculations predicted that the ejecta from 330 Type II SN must have been retained to increase the metallicity over the range observed. Norris et al. (1996; 1997) discussed self-enrichment, but preferred the merger hypothesis. Hughes & Wallerstein (2000) found evidence that ω Cen experienced a period of self-enrichment which lasted several billion years; their results from Strömgren photometry were confirmed by Hilker & Richtler (2000), with both studies concluding that there was an age spread as well a metallicity spread. From these studies, it is clear that the more metal-rich stars are several billion years younger than the most metal-poor stars in ω Cen.

Not only does ω Cen have a spread in [Fe/H], there is also a well-known variation in the CN-abundances (Smith et al. 2000, and references therein). Another interesting question arises when we consider if the surfaces of the stars could have been polluted by material from evolved cluster members (Ventura et al. 2001).

In this article, we will describe briefly the results from Hughes & Wallerstein (2000), which dealt with Strömgren photometry of stars at the main-sequence turn-off (MSTO). We will then turn to the results presented in Hughes, Wallerstein, & van Leeuwen (2002) on the MSTO and more evolved stars, including broadband photometry.

The study carried out by Hughes & Wallerstein (2000) on the MSTO stars showed that objects with high metallicities ($-1.2 < [Fe/H] < -0.5$) had an average age of 10 Gyr, those in the middle group ($-1.6 < [Fe/H] < -1.2$) were about 12 Gyr old, and the most metal-poor stars ($-2.2 < [Fe/H] < -1.6$) were 12-14 Gyr in age. The distance modulus used was $m - M = 13.77$ with $E(B - V) = 0.15$). The Strömgren data and VandenBerg et al.’s (2000) isochrones showed no evidence for a merger in the metallicity distribution (i.e., there was no strong bimodality).

Hughes & Wallerstein (2000) also took data of a single-metallicity cluster, NGC 6397, deliberately making the data noisy. This was achieved by only taking a few 900-second *vby* exposures of the cluster and not coadding the frames. By this method, the distribution of the metallicities was broadened (derived from the photometry), and we used this data set for comparison rather than simulating data. Using the real data gave a way of forcing a cluster with no known metallicity variation or age spread to mimic the width of the distribution of ω Cen. Hughes & Wallerstein (2000) confirmed that NGC 6397 was a single-metallicity cluster with no discernible age spread, whereas ω Cen had an average

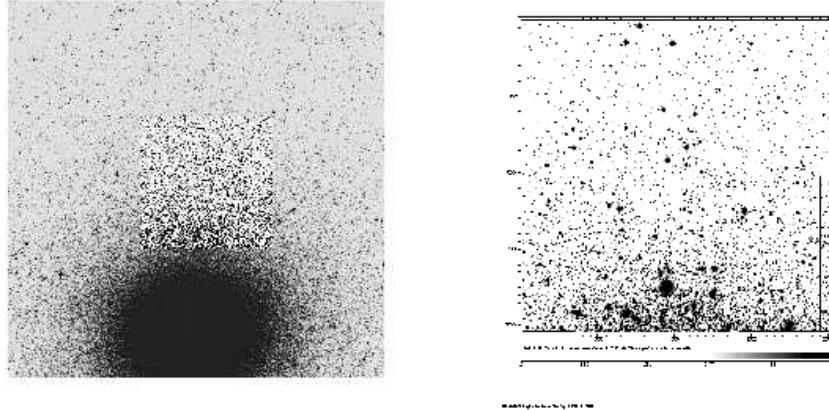


Figure 1. **a:** DSS image of ω Cen with the 300s I-exposure identified (area 13.5 square-arcminutes). Here north is up and east is to the left. **b:** The 300s I-image of the field used in this study is shown with the pixel scale; the image has a resolution of 0''.396 per pixel.

metallicity of $[Fe/H] = -1.5$, a metallicity range of $-2.5 < [Fe/H] < -0.5$, and an apparent age spread of 3-5 Gyr.

2. Observations

We observed one field $\sim 25'$ north of the cluster center ~ 90 times, at $vbyBI$ wavelengths, transforming y to V . The seeing at CTIO in May 1996 was $1 - 1''.5$ and the plate scale was 0''.396 per pixel on the Tek 2048 #3 chip. The field size is $13''.5 \times 13''.5$, but the Strömgren filters used at the time were 2×2 inches (instead of 3×3 inches for the broadband filters) and caused vignetting, thus the effective field of view was reduced to about $12''.0 \times 12''.0$. This field was selected to be far enough away from the cluster center for the 0.9-m telescope and the DAOPHOT software to be able to cope with the crowding. In addition, we examined the IRAS sky flux images at $100\mu\text{m}$ and $60\mu\text{m}$ (Wood & Bates 1993), and tried to avoid any regions which appeared to have variable extinction. We took exposures in a field away from the cluster (about 1° west of our cluster field, at approximately the same galactic latitude) to subtract field stars statistically.

We obtained $vbyBI$ photometry on 2554 sources in the on-cluster field, and 1739 stars survived the cleaning process (for details, see Hughes & Wallerstein 2000). The whole data set is shown in Figure 2.

It is almost impossible to recognize an age-abundance correlation among the giants or RR Lyrae stars. However, where the isochrones separate and flatten out at the MSTO, we can see if the more metal-rich stars form later, as would

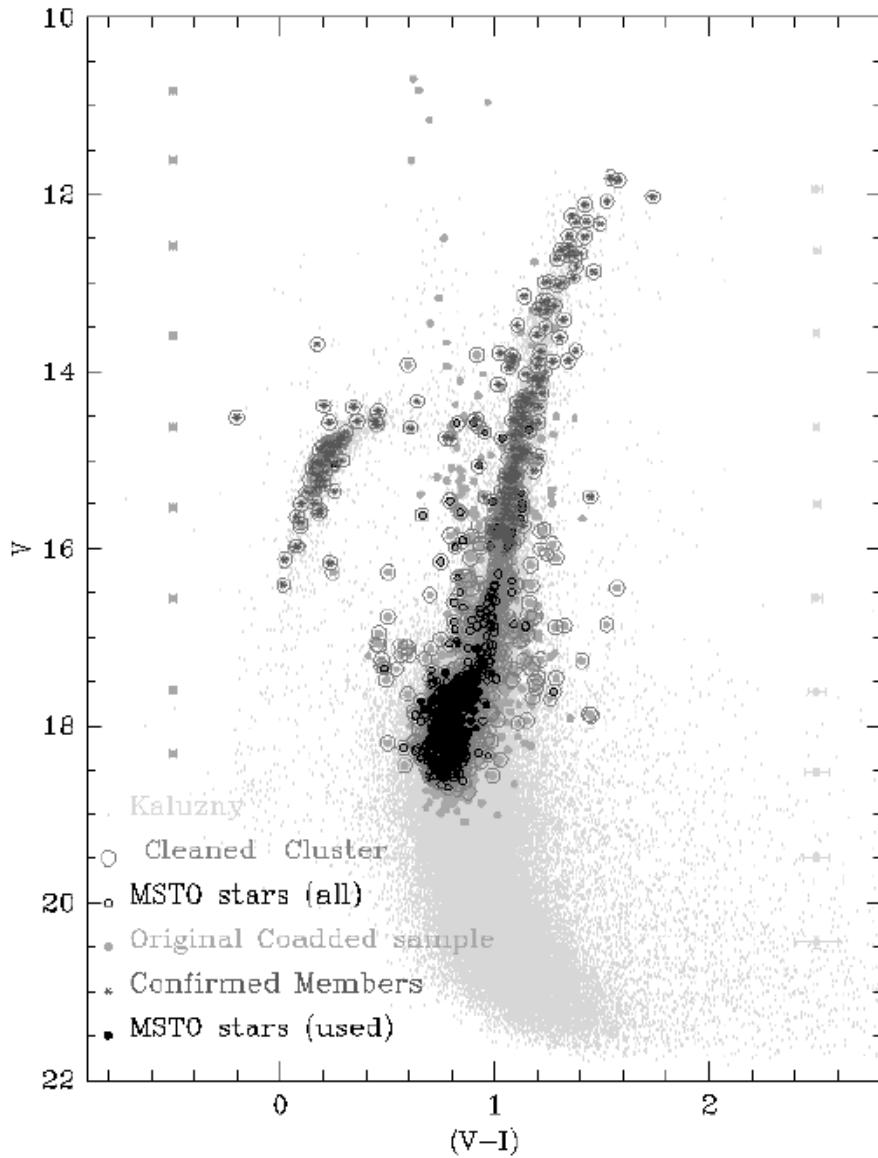


Figure 2. Color-magnitude diagram of our sample, along with the data of Kaluzny et al. (1996; 1997a; 1997b). Our uncertainties are smaller because of the coadding process, and the fact that we were not trying to get as faint as possible. The aim was to obtain better than 1.5% photometry at the MSTO and brighter. The confirmed cluster members were identified from the radial velocity and proper motion data of van Leeuwen et al. (2001).

be expected from self-enrichment (Morgan & Lake 1989). Figure 3 shows that if there was a metallicity spread, and no age spread, the more metal-rich stars should fall to the red side of the color-magnitude diagram (CMD).

Two colors (or color differences) should be measured, one that reveals the age and one that correlates with $[Fe/H]$. One of the photometric systems well designed for this purpose is the Strömgren system. In principal, the use of the $uvby$ system is very simple, but in practice, the accuracy of the data is vital to the success of the experiment.

Ideally, it would be desirable to observe in the u -band to obtain the c_1 -index (which measures surface gravity), but it was not practical at the 0.9-m telescope because of the transmission of the filter and the faintness of many of our targets in this region. The color $(b - y)$ is sensitive to temperature; $(v - b)$ is sensitive to line-blanketing. The metallicity index $m_1 = (v - b) - (b - y)$ is thus the line-blanketing at a given T_{eff} , and correlates with metallicity. The interstellar reddening is nearly the same in $(v - b)$ and $(b - y)$, making m_1 almost insensitive to interstellar extinction.

We define:

$$E(m_1) = -0.32E(b - y);$$

$$m_0 = m_1 + 0.32E(b - y);$$

which is the reddening-free metallicity index, where

$$E(b - y) = 0.7E(B - V).$$

In the range of $-0.5 < [Fe/H] - 2.0$, the slope of the m_1 vs. $[Fe/H]$ relation is $\Delta[Fe/H] = .056\Delta m_1$. Hence, we expect a total range in m_1 of 0.125 *mag*. The models of VandenBerg & Bell (1985), and VandenBerg et al. (2000) show that, at the main-sequence turn-off, the color-magnitude diagram is nearly vertical. The color index $(b - y)$ is sensitive to age with a slope given by $\Delta(b - y) = 0.010\Delta age(Gyrs)$ between 10 and 15 Gyrs. This is rather small to detect with sufficient accuracy, but the color $(B - I)$ in the broadband filters of the BVRI system shows a sensitivity of 0.025 for each Gyr, which is much easier to recognize. The effect of abundance on $(B - I)$ can be compensated for from the tables of VandenBerg et al. (2000) once the metallicity is established from the m_1 -index. We had thought that the best broadband index to obtain was $(B - I)$, but it became obvious that CN was affecting the B-magnitudes of many stars. The advantage of the longer baseline in temperature is offset by the dependence on chemical composition.

The Harris B-filter extends from 5000Å to about 3700Å, hence many features are included. In particular, many metallic lines are found here, Ca H and K, and also CN(3883Å and 4216Å) and CH(4300Å). Suntzeff (1981) looked at giants in M3 and M13, and found that asymptotic giant branch (AGB) stars have C-abundances down by a factor for 2 compared with the subgiant branch at the same temperature. In M13, almost all AGB stars and the tip of the red giant branch (RGB), are CN-poor, which would make them look metal-poor on the m_0 vs. $(b - y)_0$ diagrams. If CN is weak, the stars will look like they have weak metal lines across the B- and v-bands. It is for these reasons that we have chosen to use $(V - I)$ as the temperature index, and have converted y to the V-band (see Hughes & Wallerstein 2000, and references therein).

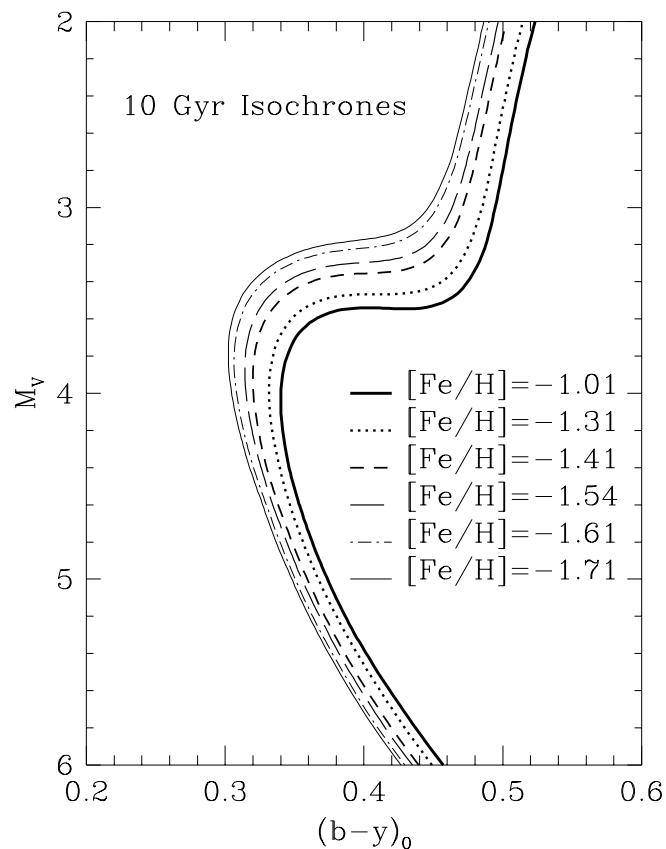


Figure 3. Color-magnitude diagram showing the effect of varying the input metallicity on the color of the main-sequence turn-off. We show the 10 Gyr isochrones for various metallicities from VandenBerg et al. (2000). If stars are the same age, the redder objects should be more metal-rich.

Before converting photometric indices to metallicities, and finding ages from the model grids of VandenBerg et al. (2000), we have to find a distance modulus and extinction for the cluster. Currently, there is a discrepancy between distances derived from different methods. At one extreme, some favor an apparent distance modulus of $m - M = 14.10$, $E(B - V) = 0.12$ from RRab stars (Rey et al. 2000); Thompson et al. (2001) find $m - M = 14.05$ and $E(B - V) = 0.13$ from model fits to the eclipsing binary, OGLE 17, which gives an absolute distance modulus of $m - M = 13.65$; and Caputo et al. (2000) find the pulsational distance to ω Cen from C-type RR Lyrae variables to be $m - M = 14.01 \pm 0.12$. Lub (2002) looked at horizontal branch (HB) stars in the cluster and determined $E(B - V) = 0.11 \pm 0.01$. He also noted that HI measurements imply $E(B - V) = 0.12$, infrared dust measurements from IRAS and COBE give $E(B - V) = 0.14$, and ultraviolet observations from Whitney et al. (1998) show $E(B - V) = 0.15$. Observations of the star ROA 24 by Gonzalez & Wallerstein (1994) give $E(B - V) = 0.18$: this star is in our field, but was saturated on the CCD. It is possible that the discrepancy between the measurements of extinction from the UV, visual and IR means that the extinction might be anomalous in this direction. At the other end of the distance range, the proper motion study of van Leeuwen et al. (2001) give a reduced absolute distance modulus of $m - M = 13.36$.

We compared our V vs. $(V - I)$ CMD to the model grids of VandenBerg et al. (2000), where $[\alpha/Fe] = +0.3$, and $Y = 0.2352$. Isochrones and population functions can be calculated from these grids (Bergbusch and VandenBerg 2001); these models do not address gravitational settling or radiative acceleration, but they are derived using current physics. From Figure 4a, we see the best-fit to the data is $m - M = 13.57$ and $E(B - V) = 0.15$. However, we will calculate ages based on the other distance moduli to examine the effect on the age-metallicity relationship.

3. Metallicity and Age

Once we have our data set, we must convert the Strömgren photometry to a metallicity. Mayluto (1994) determined a relationship between m_1 and $[Fe/H]$ within the following ranges:

$$0.22 \leq (b - y) \leq 0.38$$

$$0.03 \leq m_1 \leq 0.22$$

$$0.17 \leq c_1 \leq 0.58$$

$$-3.5 \leq [Fe/H] \leq 0.2$$

In all these relationships, the authors assume that the m_1 and $(b - y)$ values are unreddened. We define:

$$(B - Y) = ((b - y) - 0.22)/0.16 + 1;$$

$$M_1 = (m_1 - 0.03)/0.19 + 1;$$

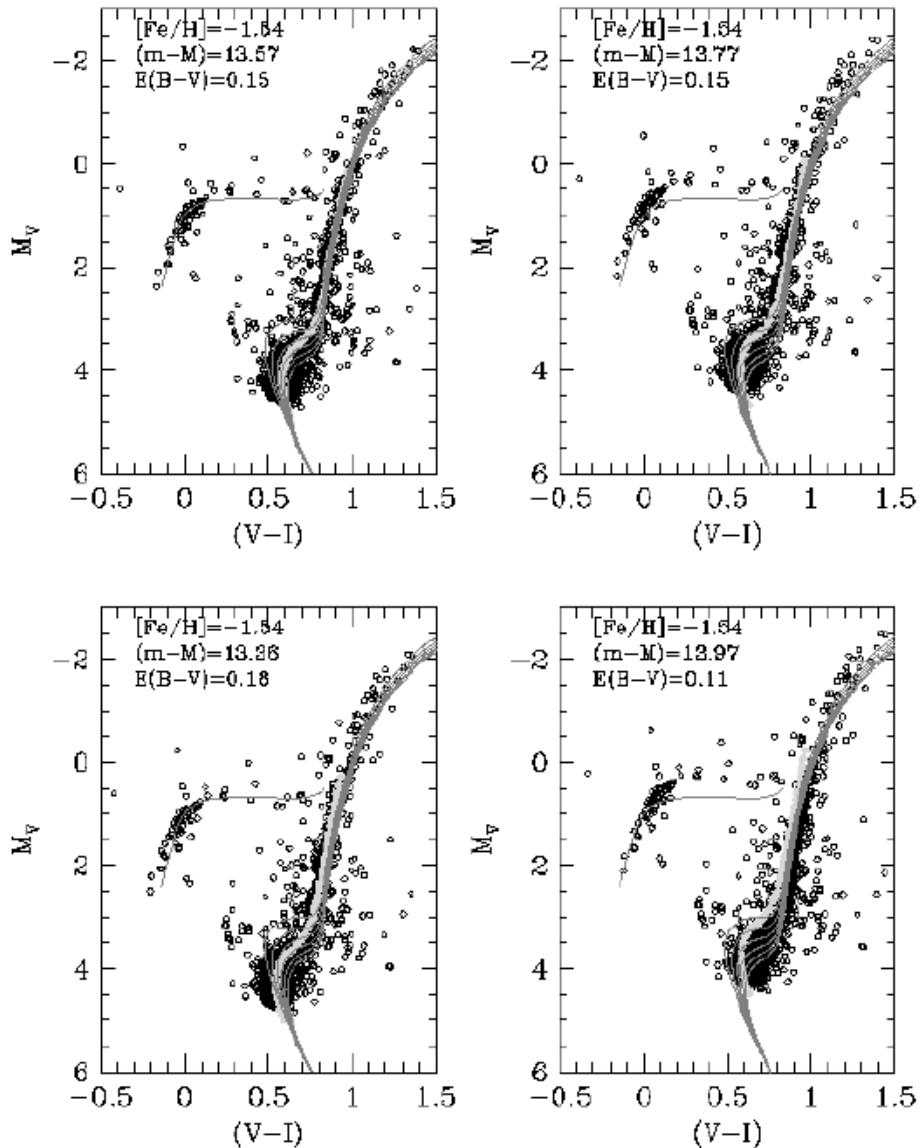


Figure 4. Using our “cleaned” cluster members, we compare the fiducial main-sequence and shape of the horizontal branch to the models of VandenBerg et al. (2000). The best fit to our data is **a**: $m - M = 13.57$ and $E(B - V) = 0.15$. Also shown are **b**: $m - M = 13.77$ with $E(B - V) = 0.15$; **c**: $m - M = 13.36$ with $E(B - V) = 0.18$; and **d**: $m - M = 13.97$ with $E(B - V) = 0.11$.

and use

$$\begin{aligned}[Fe/H] = & 5.7071(B - Y)M_1 - 49.9162(B - Y)\log M_1 + 7.9971(B - Y)^2\log M_1 \\ & - 0.5895(B - Y)^3 - 24.0889(1/M_1) + 14.6747\end{aligned}$$

Malyuto (1994) determined an uncertainty of $\sigma_{[Fe/H]} = 0.147$.

Grebel and Richtler (1992) determined another calibration for giants:

$$[Fe/H] = \frac{m_1 + a_1(b - y) + a_2}{a_3(b - y) + a_4}$$

where $a_1 = -1.24 \pm 0.006$, $a_2 = 0.294 \pm 0.03$, $a_3 = 0.472 \pm 0.04$, $a_4 = -0.118 \pm 0.02$, and these equations again imply intrinsic (unreddened) colors.

Hilker's (2000) calibration for ω Cen takes the form:

$$[Fe/H]_{phot} = \frac{m_0 - 1.277.(b - y)_0 + 0.331}{0.324.(b - y)_0 - 0.032}$$

From Figure 5, we can see that Hilker's (2000) calibration of $[Fe/H]_{phot}$ for the RGB is a better match to all the giant branch stars in ω Cen than the Grebel and Richtler (1992) calibration. There is a discontinuity between the MSTO in their narrow color-range (Malyuto 1994), but improvements will have to wait for spectra of MSTO stars in ω Cen. The stars with $m_1 > 0.2$ which are proper-motion cluster members are assumed to be part of the "anomalous" RGB defined by Pancino et al. (2000). The derived values of $[Fe/H]_{phot}$ for MSTO stars are not very sensitive to extinction.

Figure 6 shows the effect of CN-variations on the data. The open circles in Figure 6a are likely to be the CN-rich stars. The well-known CN variations in this cluster (Smith 1987) have also been seen in other clusters such as M22 (Anthony-Twarog, Twarog & Craig 1995; Richter, Hilker & Richtler 1999). The CN-abundance spread on the RGB is thus shared by the MSTO stars, which implies the variation is either primordial or the surfaces have been contaminated by AGB-star ejecta (Ventura et al. 2001).

As we did for the data set from Hughes & Wallerstein (2000), we divided the MSTO population (the CN-normal group) into three groups: "high" with $-1.2 < [Fe/H]_{phot} < -0.5$; "medium" with $-1.6 < [Fe/H]_{phot} < -1.2$; and "low" with $(-2.2 < [Fe/H]_{phot} < -1.6)$. The latter group should be the first generation of stars formed in ω Cen. We plot the data set for the corresponding metallicities for the mean of each group in Figure 7. Most of the data is fit by the $[\alpha/Fe] = 0.3$ models, but there is a suggestion that the most metal-poor stars are better fit by the $[\alpha/Fe] = 0.6$ isochrones. The α -enhancement seems to be becoming less extreme as the metallicities approach solar values. The maximum age of the cluster also fits more comfortably within the cosmological timescales if we use the $[\alpha/Fe] = 0.6$ models for the primordial stars in the cluster.

4. Summary and Conclusions

Multi-wavelength photometry is necessary in the absence of spectroscopy because the B-, I-, and v-bands are affected by CN. ($V - I$) is the best broadband

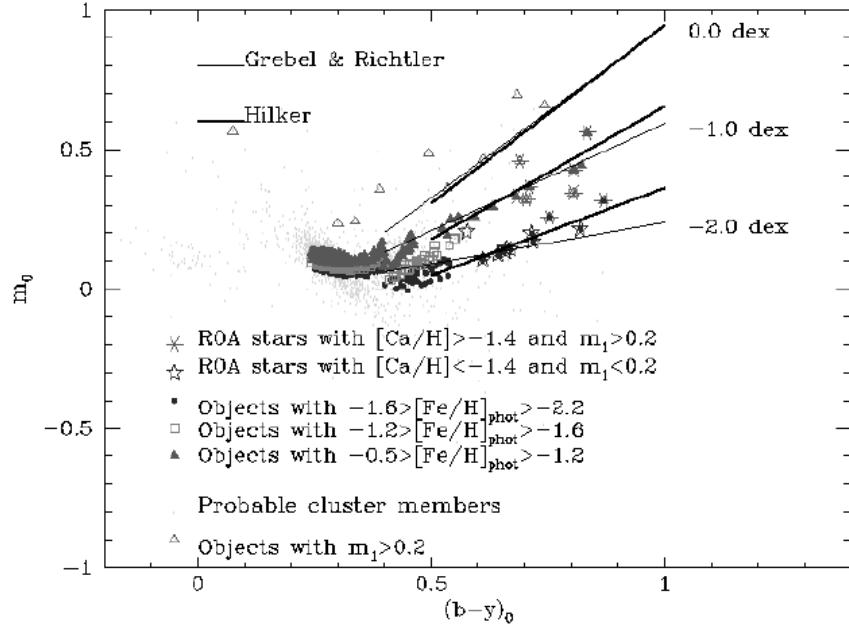


Figure 5. Color-color plot of the dereddened metallicity and temperature indices for $E(B-V) = 0.15$. The MSTO stars are selected from the color ranges specified by Malyuto (1994), and all the proper motion members are classified by Hilker's (2000) calibration.

Table 1. Population Ages (in Gyr) for Different Distances and Extinctions

$m - M$	$E(B - V)$	High $[\alpha/\text{Fe}] = 0.3$	Medium $[\alpha/\text{Fe}] = 0.3$	Low $[\alpha/\text{Fe}] = 0.3$	Low $[\alpha/\text{Fe}] = 0.6$
13.36	0.18	9.8 ± 2.2	11.5 ± 1.1	14.7 ± 2.0	13.3 ± 1.7
13.57	0.15	9.7 ± 3.0	11.9 ± 0.9	15.0 ± 2.6	12.9 ± 1.5
13.77	0.15	8.9 ± 2.1	10.1 ± 0.8	12.8 ± 1.6	10.9 ± 1.1
13.97	0.11	9.1 ± 2.0	10.1 ± 1.0	12.8 ± 1.8	11.1 ± 1.2

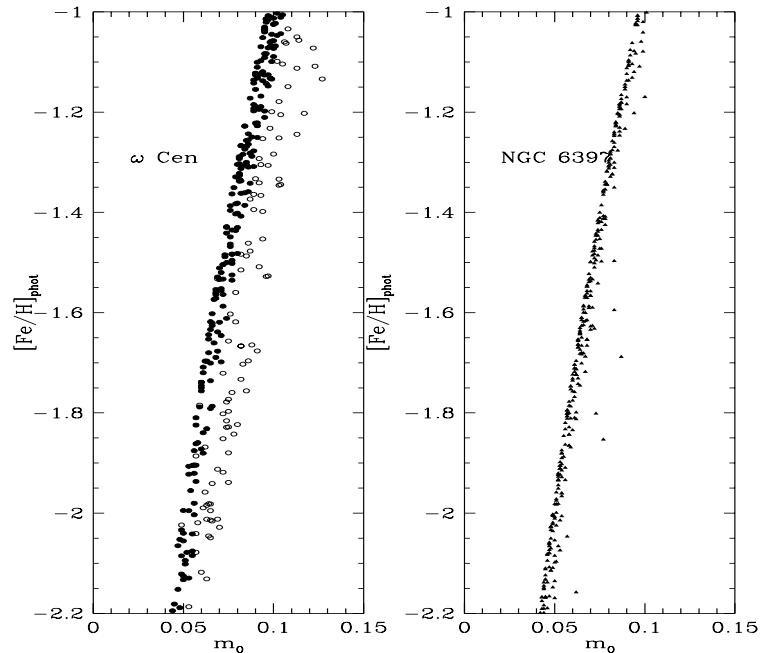


Figure 6. **a:** Plot of m_0 vs. $[Fe/H]_{phot}$ for MSTO stars in ω Cen. The filled circles are those stars that fall within the (generous) noise limits defined by the un-coadded data. By inference, stars which scatter further from this region (open circles) have done so because of anomalous chemical composition. **b:** Plot of m_0 vs. $[Fe/H]_{phot}$ for MSTO stars (and field stars that were not statistically removed) in the “noisy” data for NGC 6397.

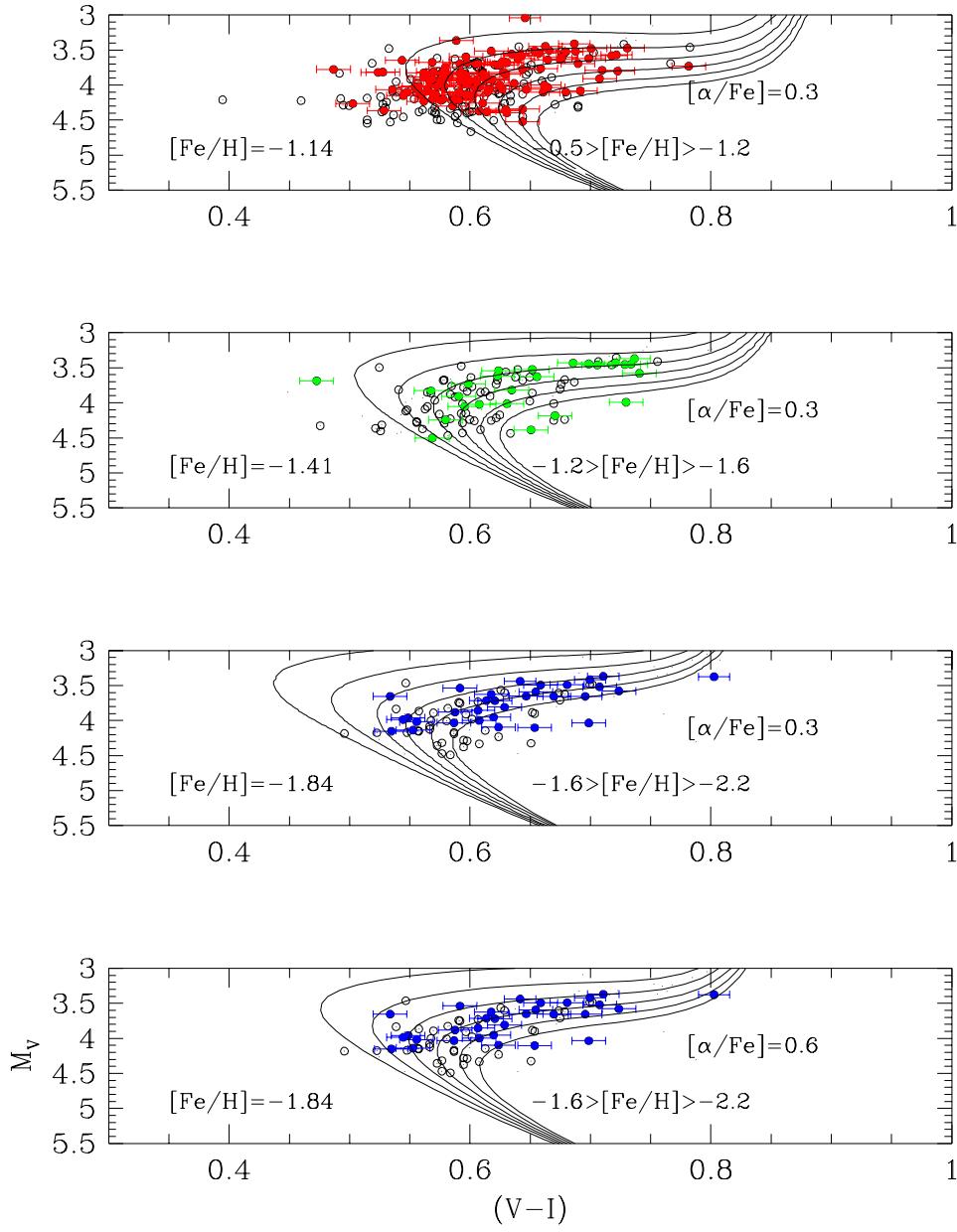


Figure 7. The CMDs are shown with the best fitting model grids from VandenBerg et al. (2000). The isochrones from left to right are 8 Gyr, 10 Gyr, 12 Gyr, 14 Gyr, 16 Gyr, and 18 Gyr. The distance modulus used is $m - M = 13.57$ and the extinction measure is $E(B - V) = 0.15$. **a:** The high metallicity group has a weighted mean age of 9.7 ± 3.0 Gyr. **b:** The medium metallicity has a weighted mean age of 11.9 ± 0.9 Gyr. **c:** The metal-poor stars have a weighted mean age of 15.0 ± 2.6 Gyr. **d:** Using the $[\alpha/Fe] = 0.6$ models (Bergbusch and VandenBerg 2001), the metal-poor stars have a weighted mean age of 12.9 ± 1.5 Gyr.

temperature index: $(B - I)$ is an indicator of T_{eff} and chemical composition. Care must be taken with the m_1 -index because it can be affected by other sources of opacity than [Fe/H] (Anthony-Twarog et al. 1995; Hughes & Wallerstein 2000; Hilker 2000).

We use the RGB to define the metallicity, and the MSTO star to find the age spread. The first and second generation of stars are separated by at least 2 Gyr, and star formation continued for $\sim 3 - 5$ Gyr. While an initial, non-uniform, composition of the stars in ω Cen is possible, it seems more likely that early generations of stars have polluted the cluster with ejecta from Type II supernovae and AGB stars.

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